Dirac quantization condition with the superconducting state

Jer Yu Lin*
Department of physics, National Taiwan University, Taipei 10617, Taiwan, Republic of China
Anthrop-Celestial Research Coordinate Institute, Tienti Church, Nantou 55542, Taiwan, Republic of China
*Email address : r89222013@ms89.ntu.edu.tw

The author argues that the Dirac quantization condition $u_0 = \frac{\hbar c}{2e}$ might imply the existence of an undiscovered electromagnetic structure which governs the quantization of the electric charge and the quantization of the magnetic flux in the superconducting state. An experimental set-up which can provide a strong evidence by predicting the discrimination between the magnetic flux generated by the positive and negative electric charge in the superconducting state is also proposed.

Although Dirac had brilliantly elucidated the quantization condition between the magnetic and the electric charge decades ago [1], the magnetic monopole is still wanted, and ongoing efforts are invested in [2].Based on Dirac´s argument [1], the magnetic monopole u_0 was assigned to be the source of the magnetic field obeying inverse-square law analogous to the role of the electric charge q in the electric field. It is obvious to notice that the vector potential needed to describe the magnetic field of u_0 can not do without a singularity [3].

In order to better solve this problem, Wu and Yang proposed a pair of vector potentials with the singularities lying along the -Z axis and the +Z axis respectively [3]:

$$\mathrm{A}_\phi^{(1)} = \frac{u_0(1-\cos\theta)}{r\sin\theta} \quad (0 \leq \theta < \pi-\varepsilon, \varepsilon \to 0), \tag{1}$$

$$\mathrm{A}_\phi^{(2)} = \frac{-u_0(1+\cos\theta)}{r\sin\theta} \quad (\varepsilon < \theta \leq \pi), \tag{2}$$

The region $(\varepsilon < \theta < \pi - \varepsilon\ )$ where overlap of these two vector potentials happens signifies a gauge transformation leading to the Dirac quantization condition [3]:

$$u_0 = \frac{\hbar c}{2e} \text{ (e} = \pm 1.602\times 10^{-19} \text{ coulomb, so in this article I define } +u_0 = \frac{\hbar c}{2e^+}, -u_0 = \frac{\hbar c}{2e^-}\text{)}, \tag{3}$$

This well chosen gauge transformation inspired me to consider the situation stated by Dirac [1] where the magnetic monopoles of opposite sign bound so strongly that we had not observed any separated ones. Namely, the magnetic fields exerted just mutually neutralize, making them become invisible. In terms of the vector potential, we can write down two pairs of vector potentials for this kind of situation:

1. $A_\phi^{(1)} = 0$, (4)

$A_\phi^{(2)} = \frac{\pm 2u_0}{r\sin\theta}$ $(\varepsilon < \theta < \pi - \varepsilon)$, (5)

$B = \nabla\times A_\phi^{(1)} = \nabla\times A_\phi^{(2)} = 0$ $(\varepsilon < \theta < \pi - \varepsilon)$, thus they are classically invisible.

$\oint A_\phi^{(2)} \cdot dl = \frac{hc}{e^\pm}$, (6)

which renders $\pm 2\pi$ to the associated Aharonov-Bohm phase of point charges [4], making them quantum mechanically invisible as well. The gauge transformation between them leads to the Dirac quantization condition.

2. $A_\phi^{(1)} = \frac{+u_0}{r\sin\theta}$ $(\varepsilon < \theta < \pi - \varepsilon)$, (7)

$A_\phi^{(2)} = \frac{-u_0}{r\sin\theta}$ $(\varepsilon < \theta < \pi - \varepsilon)$, (8)

which are constructed from the symmetrical consideration : $A_\phi^{(1)}$ is contributed by + u_0, while $A_\phi^{(2)}$ is contributed by - u_0. $B = \nabla\times A_\phi^{(1)} = \nabla\times A_\phi^{(2)} = 0$ $(\varepsilon < \theta < \pi - \varepsilon)$, thus they are classically invisible.

$\oint A_\phi^{(1)} \cdot dl = \frac{hc}{2e^+}$ (9)

and

$\oint A_\phi^{(2)} \cdot dl = \frac{hc}{2e^-}$, (10)

which render $\pm 2\pi$ to the associated Aharonov-Bohm phase of electron pairs ($q = 2e^-$) in the superconducting state, making them quantum mechanically invisible in the superconducting state. The gauge transformation between them also leads to the Dirac quantization condition.

In other words, in an area where no magnetic monopole exists, these two pairs of vector potentials still can reasonably exist except for the singularities.

Would they have any physical significance? So long as we have realized that the Dirac quantization condition can be derived from the gauge transformation represented by each of these two pairs of vector potentials, reversely, if the Dirac quantization condition is held, we can get the Aharonov-Bohm phase factor f [3]:

$\pm 2u_0 f = 1 \rightarrow f = \frac{e^\pm}{\hbar c}$ (for the point charges), (11)

$\pm u_0 f = 1 \rightarrow f = \frac{2e^\pm}{\hbar c}$ (for the superconducting state), (12)

Therefore, I assume that $\pm 2u_0(\pm u_0)$ just represents the existence of one hidden electromagnetic structure we have not discovered yet, it governs the quantization of the electric charge (the quantization of the magnetic flux in the superconducting state). The singularities in the vector potentials simply represent the need of one unknown electromagnetic coordinate transformation from the unknown electromagnetic structure represented by $\pm 2u_0(\pm u_0)$ to the one with quantized electric charges which we are familiar with.

Here I will not discuss the unknown electromagnetic coordinate transformation, but I will discuss if the unknown electromagnetic structure represented by $\pm 2u_0(\pm u_0)$ truly governs the quantization of the electric charge (the quantization of the magnetic flux in the superconducting state), could we have some experimental evidences?

1. The quantization of the electric charge : According to my assumption, the electromagnetic energy goes through the unknown electromagnetic structure represented by $\pm 2u_0$ to the quantized electric charge $e^{\pm} = \pm 1.602\times 10^{-19}$ coulomb revealed by (11) and (6).

So the famous number 1/137 may be interpretated as follows:

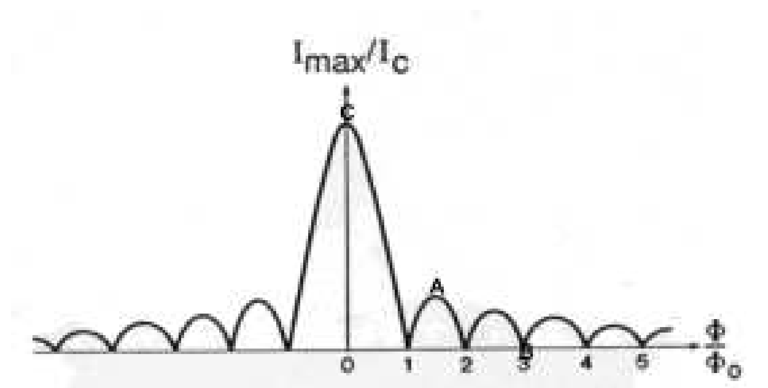
$\pm 2u_0 = \frac{\hbar c}{e^{\pm}} \rightarrow \pm 2u_0\ /137 = \mathrm{e}^{\pm}$ [1], (13)

which may represent the ratio for the electromagnetic energy quantized to the electric charge $\mathrm{e}^{\pm}$ to that still stays in the electromagnetic structure represented by $\pm 2u_0$, exactly as the coupling constant for the electromagnetic energy to an electron (positron), probably being a new interpretation of the famous number 1/137.

2. The quantization of the magnetic flux in the superconducting state: According to my assumption, the electromagnetic energy goes through the unknown electromagnetic structure represented by $\pm u_0$ to the quantized magnetic flux $\pm\Phi_0 = \frac{hc}{2e^{\pm}} = \pm 2.067 \times 10^{-15} weber$ revealed by (12) , (9) and (10).

From the present knowledge of superconductivity, the magnetic flux in the superconducting state is quantized to $\pm\Phi_0$, which had already been verified in experiments [5]. By conventional wisdom, the $\pm$ sign of the magnetic flux merely indicates the incoming or outgoing of the magnetic flux. But in my interpretation, since the $\pm$ sign of $\pm 2u_0$ discriminates the positive from the negative sign of the electric charge, the $\pm$ sign of $\pm u_0$ should discriminate the positive from the negative sign of Φ_0, namely $+\Phi_0$ generated by the positive electric charge from -Φ_0 generated by the negative electric charge. In other words, the magnetic flux generated by the positive (negative) electric charge goes through the unknown electromagnetic structure represented by $+u_0$ $(-u_0)$ to the quantized magnetic flux $+\Phi_0$ $(-\Phi_0)$. According to eq.(12), the Aharonov-Bohm phase factor for $+u_0$ $(-u_0)$ is $\frac{2e^+}{\hbar c}$ $(\frac{2e^-}{\hbar c})$, hence we have achieved an important conclusion : the Aharonov-Bohm phase factor in the superconducting state for the magnetic flux generated by the positive (negative) electric charge is $\frac{2e^+}{\hbar c}$ $(\frac{2e^-}{\hbar c})$, namely, for the associated Aharonov-Bohm phase γ of electron pairs in the superconducting state,

$\gamma = \gamma_0 + \frac{2e^-}{\hbar c} \int A \cdot dl$ (for the vector potential generated by the negative electric charge) , (14)



$\gamma = \gamma_0 + \frac{2e^+}{\hbar c}\int A \cdot dl$ (for the vector potential generated by the positive electric charge), (15)

this could be judged by the experiment.

The experiment could be done by utilizing the dc Josephson effect [6]:

$\mathrm{I} = \mathrm{I}_0 \sin\gamma$, (16)

by conventional wisdom, $\gamma = \gamma_0 + \frac{2e^-}{\hbar c}\int_1^2 A \cdot dl$ either for the vector potential generated by the negative or positive electric charge, but in my interpretation,

the Aharonov-Bohm phase shift should be opposite in sign according to (14) and (15). By intergrating eq.(16), we could get the Josephson junction diffraction

equation for a single Josephson junction [7]:

$\mathrm{I}_{\max} = I_c \frac{\sin(\pi\Phi/\Phi_0)}{\pi\Phi/\Phi_0}$, (17)

which is plotted as figure1 [7].

figure1

Therefore, if we set up a single Josephson junction under the static magnetic flux generated by the positive charge (ex. a zinc coil whose charge carrier is positive) to arrive at, say point A in figure1, then we add completely the same static magnetic flux generated by the negative charge (ex. a copper coil whose charge carrier is negative) onto it, we will arrive at point B by conventional wisdom, but according to my interpretation, we should arrive at point C.

If the above experiment could be verified, we could attain a strong evidence that there truly exists an electromagnetic structure represented by $\pm 2u_0(\pm u_0)$ which governs the quantization of the electric charge (the quantization of the

magnetic flux in the superconducting state). We might understand more about

the number 1/137 and the transition between the normal state and the

superconducting state ($\pm 2u_0 \longleftrightarrow \pm u_0$) if we go deep into the study of this structure.

*Email address : r89222013@ms89.ntu.edu.tw

[1] P. A. M. Dirac, Proc. R. Soc. London A133, 60 (1931).
[2] J. D. Jackson, Classical Electrodynamics, John Wiley and Sons, Inc., New York, 1998, P.273.
[3] T. T. Wu and C. N. Yang, Phys. Rev. D 12, 3845 (1975).
J. J. Sakurai, Modern Quantum Mechanics, Addison-Wesley Publishing Company, Inc., 1994, P.140-143.
[4] Y. Aharonov and D. Bohm, Phys. Rev. 115, 485 (1959).
[5] B. S. Deaver, Jr., and W. M. Fairbank, Phys. Rev. Lett 7, 43 (1961).
R. Doll and M. Nabauer, Phys. Rev. Lett 7, 51 (1961).
[6] B. D. Josephson, Physics Letters 1, 251 (1962).
[7] C. P. Poole, Jr., H. A. Farach, R. J. Creswick, Superconductivity, Academic Press, Inc., 1995, P.441.